

Classification of Cancer Patients with Penalized Robust Nonconvex Loss Functions

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This document presents analysis for the MAQC-II project, human breast cancer data set with penalized classification algorithms developed in Wang (2019) and implemented in R package `mpath`.

Dataset comes from the MicroArray Quality Control (MAQC) II project and includes 278 breast cancer samples with 164 estrogen receptor (ER) positive cases. The data files `GSE20194_series_matrix.txt.gz` and `GSE20194_MDACC_Sample_Info.xls` can be downloaded from <http://www.ncbi.nlm.nih.gov/geo/query/acc.cgi?token=rhojvaiwkcsaihq&acc=GSE20194>. After reading the data, some unused variables are removed. From 22283 genes, the dataset is pre-screened to obtain 3000 genes with the largest absolute values of the two-sample t-statistics. The 3000 genes are standardized.

```
# The data files below were downloaded on June 1, 2016
require("gdata")
library("mpath")
bc <- t(read.delim("GSE20194_series_matrix.txt.gz", sep = "",
  header = FALSE, skip = 80))
colnames(bc) <- bc[1, ]
bc <- bc[-1, -c(1, 2)]
### The last column is empty with variable name
### !series_matrix_table_end, thus omitted
bc <- bc[, -22284]
mode(bc) <- "numeric" ### convert character to numeric
dat1 <- read.xls("GSE20194_MDACC_Sample_Info.xls", sheet = 1,
  header = TRUE)
y <- dat1$characteristics..ER_status
y <- ifelse(y == "P", 1, -1)
table(y)
## y
## -1  1
## 114 164
res <- rep(NA, dim(bc)[2])
for (i in 1:dim(bc)[2]) res[i] <- abs(t.test(bc[, i] ~ y)$statistic)
### find 3000 largest absolute value of t-statistic
```

```

tmp <- order(res, decreasing = TRUE)[1:3000]
dat <- bc[, tmp]
### standardize variables
dat <- scale(dat)

```

Set up configuration parameters.

```

### number of replicates
nrun <- 100
### penalty type
penalty <- c("enet", "snet", "mnet")
### Smallest value for lambda, as a fraction of lambda.max, the
### smallest value for which all coefficients are zero except
### the intercept
ratio <- 0.25
type.path <- "nonactive"
nlam <- ifelse(type.path != "onestep", 30, 100)
### The training data is contaminated by randomly switching
### response variable labels at varying pre-specified
### proportions
per <- c(0, 0.05, 0.1, 0.15)
### what quantity is minimized for tuning parameter selection
tuning <- "error"
n.cores <- 5
### robust nonconvex loss function, rfamily type and logistic
type <- c("closs", "gloss", "qloss", "binomial")
### and corresponding labels
type1 <- c("Closs", "Gloss", "Qloss", "Logistic")
### and corresponding tuning parameter
s <- c(0.9, 1.01, 0.5)
mstop <- 50
plot.it <- TRUE

```

The training data contains randomly selected 50 samples with positive estrogen receptor status and 50 samples with negative estrogen receptor status, and the rest were designated as the test data. The training data is contaminated by randomly switching response variable labels at varying pre-specified proportions `per=0, 0.05, 0.1, 0.15`. This process is repeated `nrun=100` times. Robust non-convex loss functions include C-loss, G-loss and Q-loss, each with penalty LASSO, SCAD and MCP. The initial values are derived using the boosting package `bst` with `mstop=50` and `nu` provided below depending on loss function `type`. For SCAD and MCP penalty, a penalty tuning parameter `gam` is provided below. To select optimal penalization tuning parameters, we run five-fold cross-validation averaging classification errors. The classification errors and number of selected variables are tabularized and plotted with `plot.it=TRUE`. Finally, this script also contains results with penalized logistic regression using `glmreg`.

```

summary7 <- function(x) c(summary(x), sd = sd(x))
ptm <- proc.time()
for (k in (1:4)) {
  ### k controls family argument rfamily type (see above)
  if (type[k] == "gloss")
    nu <- 0.1 else nu <- 0.01
  for (j in (1:3)) {
    ### j controls argument penalty type (see above)
    gam <- ifelse(penalty[j] == "snet", 3.7, 12)
    err.m1 <- nvar.m1 <- errbest.m1 <- lambest.m1 <- matrix(NA,
      ncol = 4, nrow = nrun)
    nvarbest.m1 <- mstopcv.m1 <- matrix(NA, ncol = 4, nrow = nrun)
    colnames(err.m1) <- c("cont-0%", "cont-5%", "cont-10%",
      "cont-15%")
    colnames(mstopcv.m1) <- colnames(nvarbest.m1) <- colnames(err.m1)
    colnames(nvar.m1) <- colnames(err.m1)
    colnames(errbest.m1) <- colnames(err.m1)
    colnames(lambest.m1) <- colnames(err.m1)
    for (ii in 1:nrun) {
      set.seed(1000 + ii)
      trid <- c(sample(which(y == 1))[1:50], sample(which(y ==
        -1))[1:50])
      dtr <- dat[trid, ]
      dte <- dat[-trid, ]
      ytrold <- y[trid]
      yte <- y[-trid]
      ### number of patients/no. variables in training and test data
      dim(dtr)
      dim(dte)
      ### randomly contaminate data
      ntr <- length(trid)
      set.seed(1000 + ii)
      con <- sample(ntr)
      for (i in (1:4)) {
        ### i controls how many percentage of data contaminated, see
        ### argument per above
        ytr <- ytrold
        percon <- per[i]
        ### randomly flip labels of the samples in training set
        ### according to pre-defined contamination level
        if (percon > 0) {
          ji <- con[1:(percon * ntr)]
          ytr[ji] <- -ytrold[ji]
        }
        ### fit a model with nclreg for nonconvex loss or glmreg for
        ### logistic loss, and use cross-validation to select best
        ### penalization parameter
        if (type[k] %in% c("closs", "gloss", "qloss")) {
          dat.m1 <- nclreg(x = dtr, y = ytr, s = s[k],

```

```

        iter = 100, rfamily = type[k], penalty = penalty[j],
        lambda.min.ratio = ratio, gamma = gam, mstop.init = mstop,
        nu.init = nu, type.path = type.path, decreasing = FALSE,
        type.init = "bst")
lambda <- dat.m1$lambda[1:nlam]
set.seed(1000 + ii)
cvm1 <- cv.nclreg(x = dtr, y = ytr, nfolds = 5,
  n.cores = n.cores, parallel = TRUE, s = s[k],
  lambda = lambda, rfamily = type[k], penalty = penalty[j],
  gamma = gam, type = tuning, plot.it = FALSE,
  type.init = dat.m1$type.init, mstop.init = dat.m1$mstop.init,
  nu.init = dat.m1$nu.init, type.path = type.path,
  decreasing = dat.m1$decreasing)
err1 <- predict(dat.m1, newdata = dte, newy = yte,
  type = "error")
} else {
dat.m1 <- glmreg(x = dtr, y = (ytr + 1)/2,
  family = type[k], penalty = penalty[j], lambda.min.ratio = ratio,
  gamma = gam)
set.seed(1000 + ii)
cvm1 <- cv.glmreg(x = dtr, y = (ytr + 1)/2,
  nfolds = 5, n.cores = n.cores, parallel = TRUE,
  lambda = dat.m1$lambda, family = type[k],
  penalty = penalty[j], gamma = gam, plot.it = FALSE)
err1 <- apply((yte > -1) != predict(dat.m1,
  newx = dte, type = "class"), 2, mean)
}
optmstop <- cvm1$lambda.which
err.m1[ii, i] <- err1[optmstop]
if (ii == 1)
  varid <- names(predict(dat.m1, which = optmstop,
    type = "nonzero")) else varid <- intersect(varid, names(predict(dat.m1,
    which = optmstop, type = "nonzero")))
nvar.m1[ii, i] <- length(predict(dat.m1, which = optmstop,
  type = "nonzero"))
errbest.m1[ii, i] <- min(err1, na.rm = TRUE)
lambest.m1[ii, i] <- which.min(err1)
nvarbest.m1[ii, i] <- length(predict(dat.m1,
  which = which.min(err1), type = "nonzero"))
}
if (ii%nrnrun == 0) {
  if (type[k] %in% c("closs", "gloss", "qloss"))
    cat(paste("\nfamily ", type1[k], ", s=", s[k],
      sep = ""), "\n") else cat(paste("\nfamily ", type1[k], sep = ""),
    "\n")
  pentype <- switch(penalty[j], enet = "LASSO",
    mnet = "MCP", snet = "SCAD")
  cat("penalty=", pentype, "\n")
  if (penalty[j] %in% c("snet", "mnet"))

```

```

        cat("gamma=", gam, "\n")
        cat("common variables selected:", varid, "\n")
        cat("best misclassification error\n")
        print(round(apply(errbest.m1, 2, summary7), 4))
        cat("which lambda has best error\n")
        print(round(apply(lambest.m1, 2, summary7), 1))
        cat("number of variables selected with best error\n")
        print(round(apply(nvarbest.m1, 2, summary7),
            1))
        cat("CV based misclassification error\n")
        print(round(apply(err.m1, 2, summary7), 4))
        cat("number of variables selected by CV\n")
        print(round(apply(nvar.m1, 2, summary7), 1))
        if (plot.it) {
            par(mfrow = c(2, 1))
            boxplot(err.m1, main = "Misclassification error",
                subset = "", sub = paste(type1[k], "-", pentype,
                    sep = ""))
            boxplot(nvar.m1, main = "No. variables", subset = "",
                sub = paste(type1[k], "-", pentype, sep = ""))
        }
    }
}

}

}
##
## family Closs, s=0.9
## penalty= LASSO
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0506  0.0506   0.0449   0.0393
## 1st Qu.   0.0674  0.0674   0.0730   0.0730
## Median    0.0730  0.0730   0.0787   0.0843
## Mean      0.0746  0.0779   0.0844   0.1010
## 3rd Qu.   0.0843  0.0843   0.0899   0.1138
## Max.      0.0955  0.1461   0.1798   0.3427
## sd        0.0117  0.0151   0.0231   0.0441
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0     1.0     1.0     1.0
## 1st Qu.   1.0     1.0     1.0     1.0
## Median    9.0     12.5    8.0     10.5
## Mean     17.5    17.5    11.8    14.2
## 3rd Qu.  30.2    32.0    17.2    24.0
## Max.     58.0    50.0    44.0    46.0
## sd       18.4    16.3    12.7    13.4
## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%

```

```

## Min.      1.0      1.0      1.0      1.0
## 1st Qu.   2.0      2.0      3.0      3.0
## Median    4.0      4.0      5.0      5.0
## Mean      3.9      4.4      5.6      6.1
## 3rd Qu.   5.0      6.0      7.2      9.0
## Max.     12.0     12.0     17.0     16.0
## sd        2.1      2.7      3.1      3.5
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.     0.0506  0.0506  0.0562  0.0618
## 1st Qu.   0.0730  0.0787  0.0772  0.0899
## Median    0.0787  0.0843  0.0843  0.1039
## Mean      0.0817  0.0861  0.0922  0.1167
## 3rd Qu.   0.0899  0.0955  0.1011  0.1348
## Max.      0.1348  0.1517  0.1854  0.3427
## sd        0.0144  0.0165  0.0257  0.0455
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.   3.8      3.0      4.0      3.8
## Median    5.0      5.0      6.0      7.0
## Mean      5.2      5.1      6.0      7.2
## 3rd Qu.   7.0      7.0      7.2     10.0
## Max.     12.0     12.0     16.0     17.0
## sd        2.3      2.6      3.0      4.3
##
## family Closs, s=0.9
## penalty= SCAD
## gamma= 3.7
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.     0.0449  0.0449  0.0506  0.0506
## 1st Qu.   0.0674  0.0674  0.0674  0.0730
## Median    0.0787  0.0730  0.0787  0.0843
## Mean      0.0762  0.0794  0.0866  0.1103
## 3rd Qu.   0.0843  0.0843  0.0899  0.1348
## Max.      0.1011  0.2640  0.2753  0.2809
## sd        0.0121  0.0237  0.0340  0.0573
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.    1.0      1.0      1.0      1.0
## Median    22.5     23.5      2.5      1.0
## Mean      22.8     20.4     17.8     16.6
## 3rd Qu.   42.8     40.0     35.0     36.2
## Max.      58.0     50.0     50.0     48.0
## sd        21.7     19.2     18.1     18.8
## number of variables selected with best error

```

```

##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.   1.0      1.0      1.0      1.0
## Median    1.0      1.0      1.0      1.0
## Mean      1.0      1.0      1.0      1.2
## 3rd Qu.   1.0      1.0      1.0      1.0
## Max.      2.0      2.0      2.0      7.0
## sd        0.2      0.1      0.2      1.0
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0506  0.0449  0.0562  0.0506
## 1st Qu.   0.0730  0.0730  0.0730  0.0787
## Median    0.0787  0.0787  0.0843  0.0899
## Mean      0.0805  0.0839  0.0910  0.1150
## 3rd Qu.   0.0899  0.0899  0.0955  0.1404
## Max.      0.1067  0.2753  0.2753  0.3090
## sd        0.0115  0.0261  0.0358  0.0598
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1        1        1.0      1.0
## 1st Qu.   1        1        1.0      1.0
## Median    1        1        1.0      1.0
## Mean      1        1        1.0      1.2
## 3rd Qu.   1        1        1.0      1.0
## Max.      1        1        2.0      7.0
## sd        0        0        0.1      0.9
##
## family Closs, s=0.9
## penalty= MCP
## gamma= 12
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0449  0.0449  0.0506  0.0506
## 1st Qu.   0.0674  0.0674  0.0674  0.0730
## Median    0.0787  0.0730  0.0787  0.0843
## Mean      0.0772  0.0805  0.0871  0.1060
## 3rd Qu.   0.0843  0.0899  0.0899  0.1306
## Max.      0.1124  0.2191  0.2303  0.2584
## sd        0.0127  0.0245  0.0322  0.0508
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.   1.0      1.0      1.0      1.0
## Median    1.5     11.0     1.0      3.5
## Mean     18.1     18.7     12.1     12.5
## 3rd Qu.  35.5     35.5     21.2     25.2
## Max.     60.0     55.0     52.0     49.0
## sd      21.6     19.3     15.8     15.5

```

```

## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.    1.0      1.0      1.0      1.0
## Median     1.0      1.0      1.0      2.0
## Mean       1.4      1.4      1.7      2.6
## 3rd Qu.    2.0      2.0      2.0      4.0
## Max.       7.0      6.0      6.0     10.0
## sd         0.8      0.9      1.1      2.1
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0449  0.0449  0.0562  0.0562
## 1st Qu.    0.0730  0.0730  0.0730  0.0787
## Median     0.0843  0.0843  0.0843  0.0899
## Mean       0.0819  0.0858  0.0928  0.1158
## 3rd Qu.    0.0899  0.0899  0.0969  0.1404
## Max.       0.1124  0.2191  0.2416  0.2809
## sd         0.0129  0.0244  0.0335  0.0541
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.    1.0      1.0      1.0      1.0
## Median     1.0      1.0      1.0      1.5
## Mean       1.5      1.5      1.7      2.7
## 3rd Qu.    2.0      2.0      2.0      4.0
## Max.       7.0      7.0      8.0     11.0
## sd         1.0      1.1      1.3      2.4
##
## family Gloss, s=1.01
## penalty= LASSO
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0449  0.0506  0.0449  0.0562
## 1st Qu.    0.0674  0.0674  0.0674  0.0730
## Median     0.0730  0.0730  0.0787  0.0899
## Mean       0.0752  0.0771  0.0835  0.1037
## 3rd Qu.    0.0843  0.0843  0.0899  0.1194
## Max.       0.0955  0.1236  0.1629  0.3146
## sd         0.0120  0.0139  0.0221  0.0413
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.    1.0      1.0      2.8      7.0
## Median     15.0     20.0     18.0     25.0
## Mean       25.6     27.3     22.7     27.4
## 3rd Qu.    51.0     50.0     38.0     44.0
## Max.       83.0     76.0     78.0     77.0
## sd         26.8     25.6     21.4     22.1

```



```

## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.    2.0      2.0      3.0      4.0
## Median     4.0      4.0      6.0      7.0
## Mean       4.1      4.8      6.0      7.9
## 3rd Qu.    5.0      7.0      8.0     11.0
## Max.      10.0     17.0     21.0     21.0
## sd         2.4      3.3      3.7      4.8
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0506  0.0506  0.0618  0.0674
## 1st Qu.    0.0730  0.0787  0.0787  0.0899
## Median     0.0843  0.0843  0.0899  0.1152
## Mean       0.0833  0.0858  0.0969  0.1266
## 3rd Qu.    0.0955  0.0913  0.1025  0.1531
## Max.       0.1236  0.1629  0.2303  0.3146
## sd         0.0138  0.0176  0.0286  0.0463
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      2.0
## 1st Qu.    4.0      4.0      4.0      6.0
## Median     5.0      5.5      6.5      9.0
## Mean       5.2      6.0      7.6     10.4
## 3rd Qu.    6.2      8.0     10.0     14.0
## Max.      11.0     15.0     21.0     24.0
## sd         2.2      2.8      4.5      5.5
##
## family Gloss, s=1.01
## penalty= SCAD
## gamma= 3.7
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0562  0.0562  0.0562  0.0618
## 1st Qu.    0.0730  0.0730  0.0730  0.0787
## Median     0.0787  0.0843  0.0843  0.0955
## Mean       0.0813  0.0849  0.0944  0.1276
## 3rd Qu.    0.0899  0.0899  0.0955  0.1601
## Max.       0.1124  0.2640  0.3427  0.3371
## sd         0.0125  0.0229  0.0425  0.0715
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0      1.0      1.0      1.0
## 1st Qu.    1.0      1.0      1.0      1.0
## Median     1.0      1.0      1.0      1.0
## Mean       1.9      2.5      3.0      6.2
## 3rd Qu.    1.2      2.0      2.0      2.0
## Max.      34.0     88.0     81.0     95.0

```

```

## sd          3.7      8.9      9.8      16.4
## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.          1         1         1         1.0
## 1st Qu.        1         1         1         1.0
## Median         1         1         1         1.0
## Mean           1         1         1         1.6
## 3rd Qu.        1         1         1         1.0
## Max.           1         1         1        21.0
## sd             0         0         0         2.5
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.    0.0562  0.0562  0.0562  0.0674
## 1st Qu. 0.0730  0.0787  0.0787  0.0843
## Median  0.0787  0.0843  0.0843  0.1011
## Mean    0.0836  0.0878  0.0972  0.1330
## 3rd Qu. 0.0955  0.0955  0.1011  0.1685
## Max.    0.1124  0.2697  0.3427  0.3539
## sd      0.0128  0.0229  0.0432  0.0726
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.          1         1         1         1.0
## 1st Qu.        1         1         1         1.0
## Median         1         1         1         1.0
## Mean           1         1         1         2.1
## 3rd Qu.        1         1         1         1.0
## Max.           1         1         1        26.0
## sd             0         0         0         4.4
##
## family Gloss, s=1.01
## penalty= MCP
## gamma= 12
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.    0.0449  0.0506  0.0506  0.0506
## 1st Qu. 0.0674  0.0674  0.0674  0.0730
## Median  0.0787  0.0787  0.0787  0.0843
## Mean    0.0784  0.0803  0.0852  0.1029
## 3rd Qu. 0.0899  0.0899  0.0899  0.1292
## Max.    0.1011  0.2022  0.2135  0.3483
## sd      0.0122  0.0195  0.0271  0.0499
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.          1.0      1.0      1.0      1.0
## 1st Qu.        1.0      1.0      1.0      6.0
## Median         1.0      2.5     10.5     21.0
## Mean          20.2     23.3     20.4     27.7
## 3rd Qu.        39.8     48.2     37.0     49.5

```

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## Max.      84.0    77.0    68.0    72.0
## sd        28.8    27.3    22.0    23.3
## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0     1.0     1.0     1.0
## 1st Qu.   1.0     1.0     1.0     1.0
## Median    1.0     1.0     1.0     1.0
## Mean      1.1     1.3     1.6     2.6
## 3rd Qu.   1.0     1.0     2.0     3.0
## Max.      3.0     9.0     9.0    13.0
## sd        0.4     1.2     1.2     2.7
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0506  0.0506  0.0506  0.0618
## 1st Qu.   0.0730  0.0730  0.0730  0.0843
## Median    0.0787  0.0815  0.0843  0.1067
## Mean      0.0818  0.0856  0.0944  0.1308
## 3rd Qu.   0.0899  0.0899  0.1011  0.1756
## Max.      0.1292  0.2022  0.2640  0.3764
## sd        0.0135  0.0239  0.0353  0.0583
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0     1.0     1.0     1.0
## 1st Qu.   1.0     1.0     1.0     1.0
## Median    1.0     1.0     1.0     2.0
## Mean      1.1     1.3     2.0     4.6
## 3rd Qu.   1.0     1.0     2.0     8.0
## Max.      7.0     8.0    17.0    18.0
## sd        0.7     1.0     2.7     4.9
##
## family Qloss, s=0.5
## penalty= LASSO
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0449  0.0506  0.0449  0.0562
## 1st Qu.   0.0674  0.0674  0.0674  0.0730
## Median    0.0730  0.0730  0.0787  0.0899
## Mean      0.0748  0.0767  0.0837  0.1026
## 3rd Qu.   0.0843  0.0843  0.0899  0.1180
## Max.      0.0955  0.1180  0.1685  0.3090
## sd        0.0117  0.0136  0.0223  0.0407
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0     1.0     1.0     1.0
## 1st Qu.   1.0     1.0     3.0     8.0
## Median    15.5    21.0    18.0    25.0
## Mean      25.6    26.9    23.1    27.1
## 3rd Qu.   51.0    46.8    37.5    43.0

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## Max.      81.0   74.0   74.0   73.0
## sd        27.1   24.8   21.4   20.9
## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0    1.0    1.0    1.0
## 1st Qu.   2.0    2.0    3.0    4.0
## Median    4.0    4.0    6.0    7.0
## Mean      4.3    4.9    6.2    7.9
## 3rd Qu.   6.0    7.0    8.2    11.0
## Max.     11.0   17.0   16.0   20.0
## sd        2.5    3.4    3.6    4.8
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0506 0.0506 0.0618 0.0674
## 1st Qu.   0.0730 0.0772 0.0829 0.0899
## Median    0.0815 0.0843 0.0899 0.1180
## Mean      0.0823 0.0851 0.0973 0.1252
## 3rd Qu.   0.0899 0.0913 0.1067 0.1517
## Max.      0.1236 0.1517 0.2191 0.3090
## sd        0.0138 0.0162 0.0277 0.0448
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0    2.0    1.0    2.0
## 1st Qu.   4.0    4.0    4.0    6.0
## Median    5.0    6.0    6.0    9.5
## Mean      5.8    6.2    7.8    10.6
## 3rd Qu.   7.0    8.0    10.0   14.0
## Max.     11.0   14.0   24.0   27.0
## sd        2.3    2.7    5.1    6.0
##
## family Qloss, s=0.5
## penalty= SCAD
## gamma= 3.7
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0562 0.0562 0.0562 0.0618
## 1st Qu.   0.0730 0.0730 0.0730 0.0772
## Median    0.0787 0.0843 0.0843 0.0927
## Mean      0.0810 0.0856 0.0938 0.1219
## 3rd Qu.   0.0899 0.0899 0.0955 0.1517
## Max.      0.1124 0.2640 0.3427 0.3258
## sd        0.0126 0.0251 0.0425 0.0664
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0    1.0    1.0    1.0
## 1st Qu.   1.0    1.0    1.0    1.0
## Median    1.0    1.0    1.0    1.0
## Mean      2.0    1.9    1.9    2.8

```

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## 3rd Qu.      2.0      2.0      2.0      2.0
## Max.        33.0     15.0     11.0     53.0
## sd          3.5      2.0      1.9      6.8
## number of variables selected with best error
##           cont-0% cont-5% cont-10% cont-15%
## Min.         1        1        1        1.0
## 1st Qu.      1        1        1        1.0
## Median       1        1        1        1.0
## Mean         1        1        1        1.5
## 3rd Qu.      1        1        1        1.0
## Max.         1        1        1        20.0
## sd           0        0        0        2.3
## CV based misclassification error
##           cont-0% cont-5% cont-10% cont-15%
## Min.      0.0562  0.0562  0.0562  0.0674
## 1st Qu.   0.0730  0.0787  0.0772  0.0843
## Median    0.0787  0.0843  0.0843  0.0955
## Mean      0.0833  0.0892  0.0974  0.1266
## 3rd Qu.   0.0913  0.0955  0.1011  0.1531
## Max.      0.1124  0.2753  0.3427  0.3315
## sd        0.0129  0.0255  0.0435  0.0663
## number of variables selected by CV
##           cont-0% cont-5% cont-10% cont-15%
## Min.         1        1        1        1.0
## 1st Qu.      1        1        1        1.0
## Median       1        1        1        1.0
## Mean         1        1        1        1.8
## 3rd Qu.      1        1        1        1.0
## Max.         1        1        1        26.0
## sd           0        0        0        3.6
##
## family Qloss, s=0.5
## penalty= MCP
## gamma= 12
## common variables selected:
## best misclassification error
##           cont-0% cont-5% cont-10% cont-15%
## Min.      0.0449  0.0506  0.0506  0.0506
## 1st Qu.   0.0674  0.0674  0.0674  0.0730
## Median    0.0787  0.0787  0.0787  0.0843
## Mean      0.0781  0.0798  0.0851  0.1016
## 3rd Qu.   0.0899  0.0899  0.0899  0.1292
## Max.      0.1067  0.1966  0.2079  0.3258
## sd        0.0122  0.0191  0.0268  0.0483
## which lambda has best error
##           cont-0% cont-5% cont-10% cont-15%
## Min.         1.0      1.0      1.0      1.0
## 1st Qu.      1.0      1.0      1.0      1.0
## Median       1.0      4.5      7.0     23.0

```

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## Mean      21.9    23.0    19.3    26.6
## 3rd Qu.   51.2    49.0    36.2    48.0
## Max.      82.0    77.0    75.0    69.0
## sd        29.4    26.6    22.6    22.8
## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0     1.0     1.0     1.0
## 1st Qu.   1.0     1.0     1.0     1.0
## Median    1.0     1.0     1.0     2.0
## Mean      1.2     1.4     1.8     2.9
## 3rd Qu.   1.0     1.0     2.0     3.0
## Max.      2.0     9.0    12.0    15.0
## sd        0.4     1.1     1.6     3.0
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0562  0.0506  0.0562  0.0618
## 1st Qu.   0.0730  0.0730  0.0730  0.0843
## Median    0.0787  0.0787  0.0843  0.1011
## Mean      0.0818  0.0849  0.0931  0.1272
## 3rd Qu.   0.0899  0.0899  0.0955  0.1643
## Max.      0.1292  0.2022  0.2640  0.3876
## sd        0.0133  0.0214  0.0339  0.0584
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0     1.0     1.0     1.0
## 1st Qu.   1.0     1.0     1.0     1.0
## Median    1.0     1.0     1.0     2.0
## Mean      1.3     1.5     2.2     5.0
## 3rd Qu.   1.0     1.2     2.0     9.0
## Max.      7.0     9.0    17.0    18.0
## sd        0.8     1.2     2.8     5.2
##
## family Logistic
## penalty= LASSO
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0449  0.0449  0.0562  0.0562
## 1st Qu.   0.0660  0.0674  0.0730  0.0843
## Median    0.0730  0.0787  0.0843  0.1124
## Mean      0.0729  0.0811  0.0939  0.1176
## 3rd Qu.   0.0787  0.0899  0.1067  0.1461
## Max.      0.1011  0.1685  0.1854  0.3034
## sd        0.0114  0.0200  0.0297  0.0413
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      2.0     6.0     1.0     5.0
## 1st Qu.   2.0    21.0    19.0    19.0
## Median    51.5    34.0    32.5    32.0

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## Mean      45.4    40.7    37.3    39.1
## 3rd Qu.   73.2    55.0    54.5    55.5
## Max.      99.0   100.0   100.0   100.0
## sd        33.2    24.3    24.5    25.8
## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      1.0     1.0     1.0     1.0
## 1st Qu.   1.0     1.0     2.0     3.0
## Median    3.0     3.0     3.0     6.0
## Mean      3.8     4.0     5.7     8.9
## 3rd Qu.   5.0     5.0     7.0    11.5
## Max.     15.0    16.0    29.0    32.0
## sd        3.2     3.5     5.8     8.1
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0562  0.0618  0.0618  0.0843
## 1st Qu.   0.0772  0.0885  0.1011  0.1292
## Median    0.0843  0.1011  0.1236  0.1573
## Mean      0.0851  0.1058  0.1290  0.1567
## 3rd Qu.   0.0955  0.1180  0.1461  0.1812
## Max.      0.1292  0.2079  0.2191  0.3371
## sd        0.0143  0.0262  0.0356  0.0406
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.      4.0     4.0     3.0     4.0
## 1st Qu.   9.0    13.0    13.0    13.0
## Median   11.5    16.0    19.0    20.0
## Mean     11.7    16.5    18.6    19.4
## 3rd Qu.  15.0    20.0    24.0    26.0
## Max.     20.0    29.0    33.0    33.0
## sd        3.6     5.2     7.3     7.7
##
## family Logistic
## penalty= SCAD
## gamma= 3.7
## common variables selected:
## best misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0449  0.0449  0.0562  0.0562
## 1st Qu.   0.0674  0.0674  0.0730  0.0843
## Median    0.0730  0.0787  0.0843  0.1124
## Mean      0.0737  0.0811  0.0940  0.1176
## 3rd Qu.   0.0843  0.0899  0.1067  0.1461
## Max.      0.1011  0.1685  0.1854  0.3034
## sd        0.0115  0.0200  0.0297  0.0413
## which lambda has best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      2.0     6.0     1.0     5.0
## 1st Qu.   2.0    21.0    19.0    19.0

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## Median      49.5    34.0    32.0    32.0
## Mean        44.3    39.7    36.7    39.1
## 3rd Qu.     73.8    53.2    52.5    55.5
## Max.        100.0   98.0    100.0   100.0
## sd          34.4    23.4    24.2    25.8
## number of variables selected with best error
##           cont-0% cont-5% cont-10% cont-15%
## Min.         1.0     1.0     1.0     1.0
## 1st Qu.       1.0     1.0     2.0     3.0
## Median        2.5     3.0     3.0     6.0
## Mean          3.4     3.8     5.5     8.9
## 3rd Qu.       5.0     4.0     7.0    11.5
## Max.         15.0    16.0    29.0    32.0
## sd           2.8     3.4     5.6     8.1
## CV based misclassification error
##           cont-0% cont-5% cont-10% cont-15%
## Min.         0.0562  0.0618  0.0618  0.0843
## 1st Qu.      0.0772  0.0843  0.1011  0.1292
## Median       0.0843  0.1011  0.1236  0.1573
## Mean         0.0846  0.1051  0.1289  0.1566
## 3rd Qu.     0.0899  0.1138  0.1461  0.1812
## Max.         0.1292  0.2079  0.2191  0.3371
## sd           0.0127  0.0264  0.0359  0.0409
## number of variables selected by CV
##           cont-0% cont-5% cont-10% cont-15%
## Min.         2.0     3.0     3.0     4.0
## 1st Qu.       6.0    12.0    12.8    13.0
## Median        8.0    15.0    19.0    20.0
## Mean          9.1    15.3    18.5    19.1
## 3rd Qu.      12.0    20.0    24.0    25.0
## Max.         20.0    29.0    33.0    33.0
## sd           4.3     5.8     7.3     7.6
##
## family Logistic
## penalty= MCP
## gamma= 12
## common variables selected:
## best misclassification error
##           cont-0% cont-5% cont-10% cont-15%
## Min.         0.0449  0.0449  0.0506  0.0562
## 1st Qu.      0.0674  0.0674  0.0730  0.0843
## Median       0.0730  0.0787  0.0843  0.1152
## Mean         0.0743  0.0798  0.0936  0.1184
## 3rd Qu.     0.0843  0.0899  0.1011  0.1461
## Max.         0.1011  0.1685  0.1966  0.2921
## sd           0.0120  0.0199  0.0314  0.0418
## which lambda has best error
##           cont-0% cont-5% cont-10% cont-15%
## Min.         2.0     6.0     2.0     5.0

```

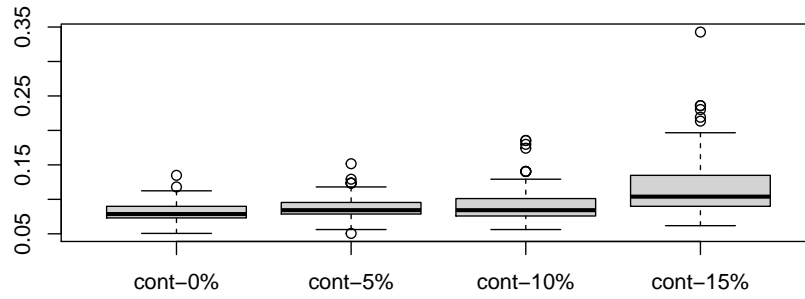


```

## 1st Qu.      2.0    24.5    20.0    22.5
## Median      55.5    41.0    38.0    34.0
## Mean       50.0    45.3    38.8    41.4
## 3rd Qu.    84.0    63.2    53.8    53.0
## Max.      98.0    97.0    99.0   100.0
## sd       35.4    24.7    24.6    26.1
## number of variables selected with best error
##          cont-0% cont-5% cont-10% cont-15%
## Min.         1.0     1.0     1.0     1.0
## 1st Qu.      1.0     1.0     2.0     3.0
## Median      2.0     3.0     3.0     5.0
## Mean        3.0     3.6     5.0     8.1
## 3rd Qu.     4.0     5.0     7.0    11.0
## Max.       10.0    16.0    17.0    28.0
## sd         2.2     3.1     4.3     7.1
## CV based misclassification error
##          cont-0% cont-5% cont-10% cont-15%
## Min.      0.0562  0.0618  0.0562  0.0730
## 1st Qu.   0.0730  0.0843  0.1011  0.1292
## Median   0.0843  0.0955  0.1208  0.1629
## Mean     0.0844  0.1032  0.1289  0.1608
## 3rd Qu.  0.0955  0.1124  0.1517  0.1854
## Max.     0.1292  0.2360  0.2360  0.3315
## sd      0.0132  0.0268  0.0397  0.0443
## number of variables selected by CV
##          cont-0% cont-5% cont-10% cont-15%
## Min.         2.0     2.0     1.0     2.0
## 1st Qu.      6.0     9.0     8.0    10.0
## Median      8.0    12.0    15.0    15.0
## Mean        8.4    11.9    14.1    15.1
## 3rd Qu.    11.0    15.0    20.0    21.0
## Max.       17.0    23.0    27.0    29.0
## sd         3.3     4.5     6.7     6.7
print(proc.time() - ptm)
##          user      system  elapsed
## 32438.825   318.385 53197.438

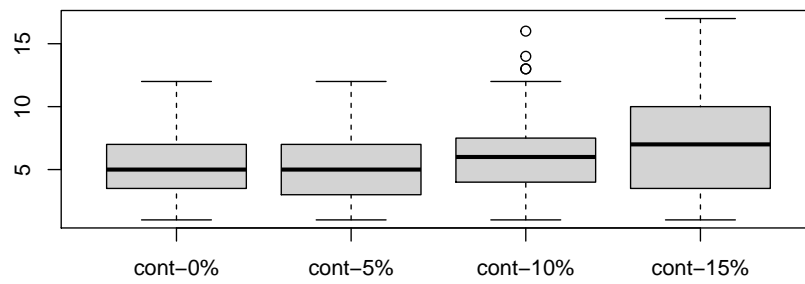
```

Misclassification error



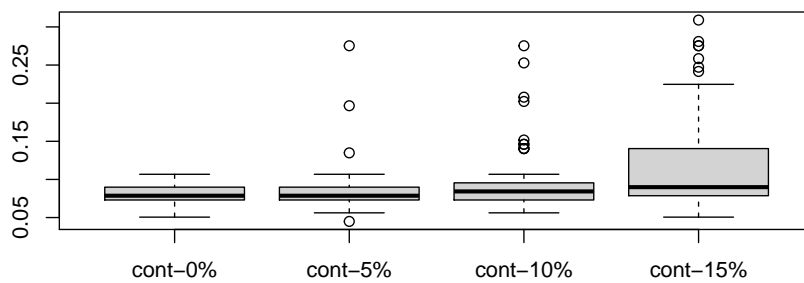
Closs-LASSO

No. variables



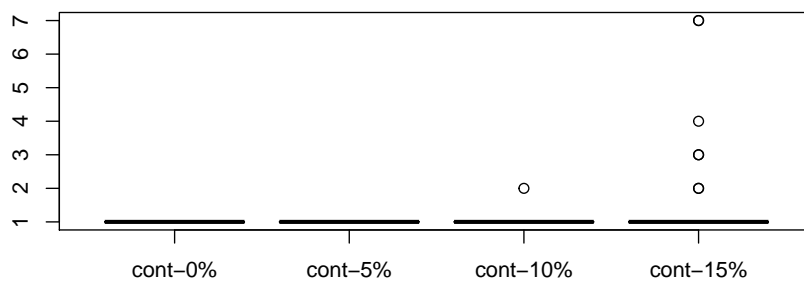
Closs-LASSO

Misclassification error



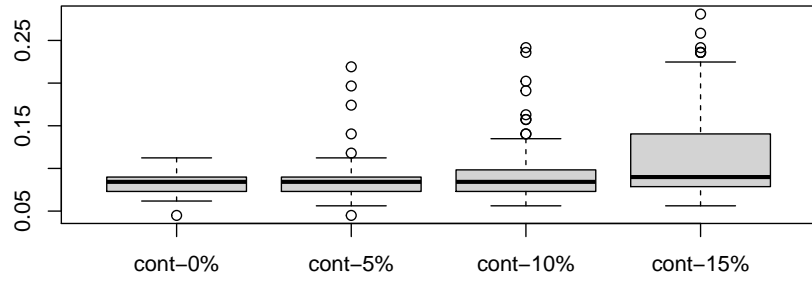
Closs-SCAD

No. variables



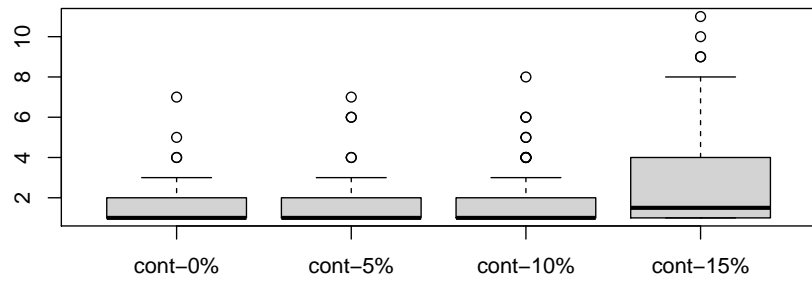
Closs-SCAD

Misclassification error



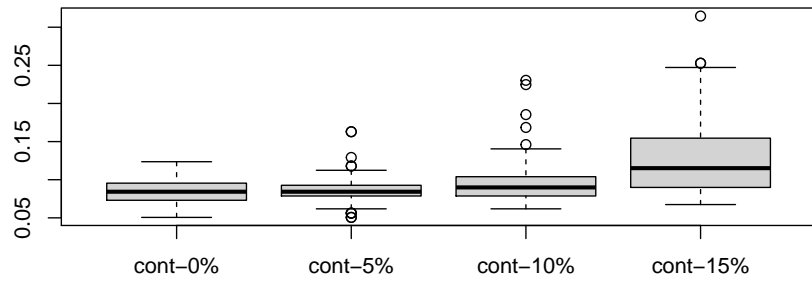
Cross-MCP

No. variables



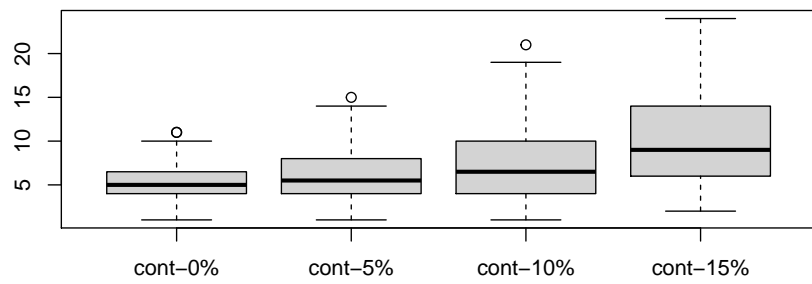
Cross-MCP

Misclassification error



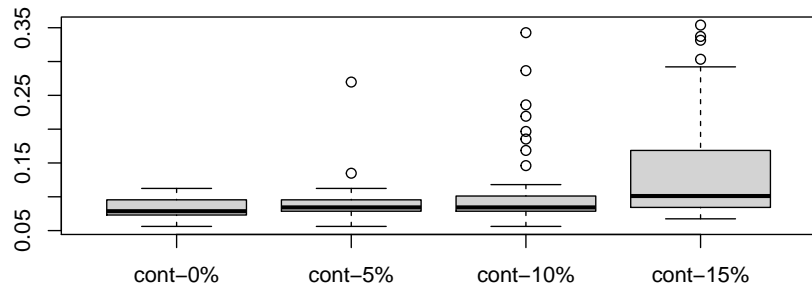
Gloss-LASSO

No. variables



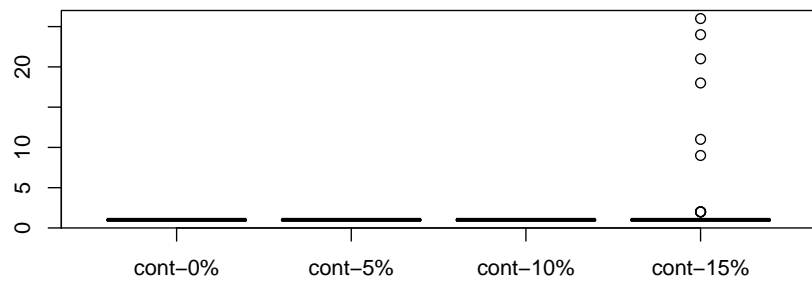
Gloss-LASSO

Misclassification error



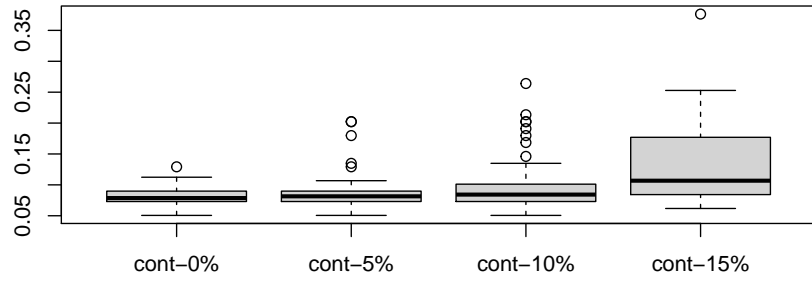
Gloss-SCAD

No. variables



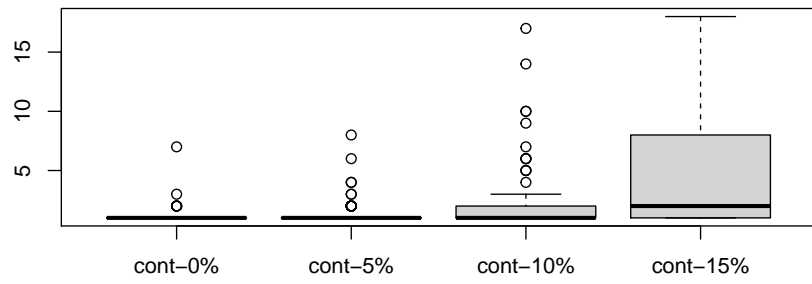
Gloss-SCAD

Misclassification error



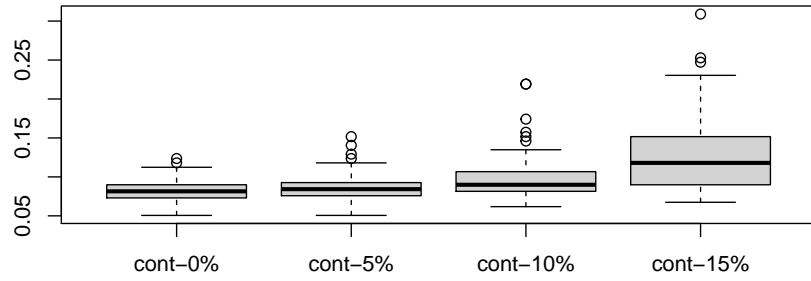
Gloss-MCP

No. variables



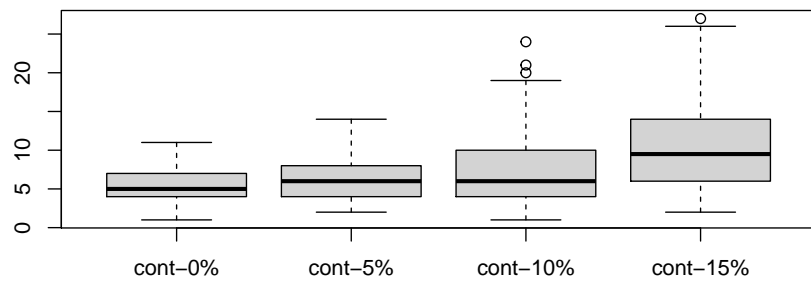
Gloss-MCP

Misclassification error



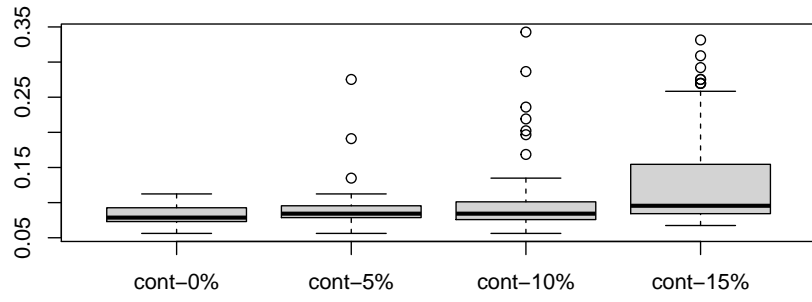
Qloss-LASSO

No. variables



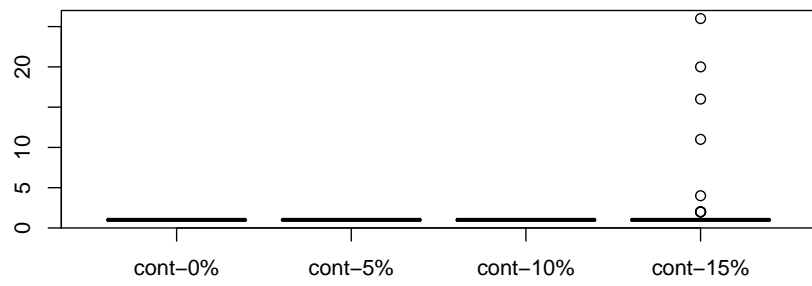
Qloss-LASSO

Misclassification error



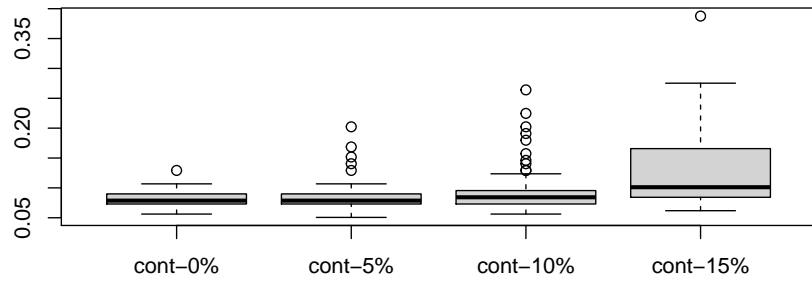
Qloss-SCAD

No. variables



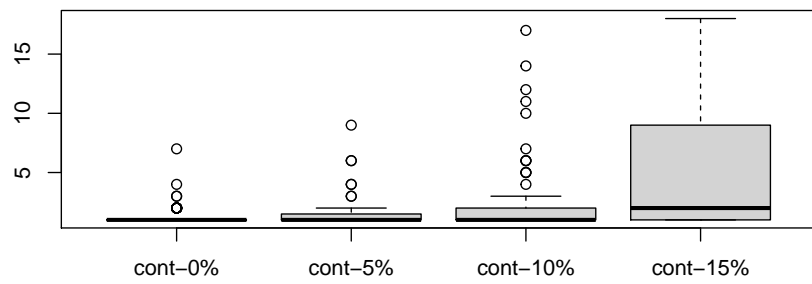
Qloss-SCAD

Misclassification error



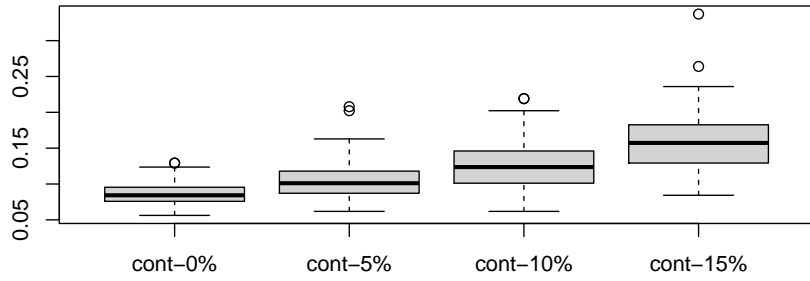
Qloss-MCP

No. variables



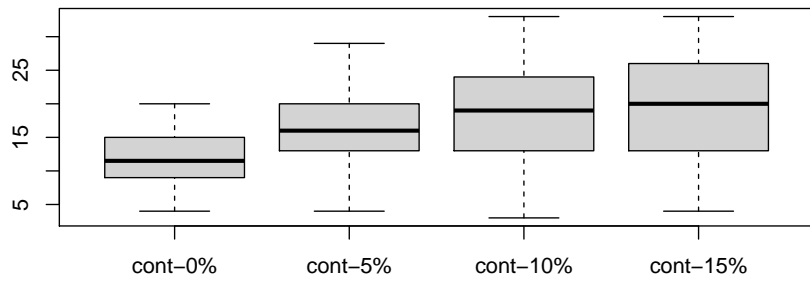
Qloss-MCP

Misclassification error



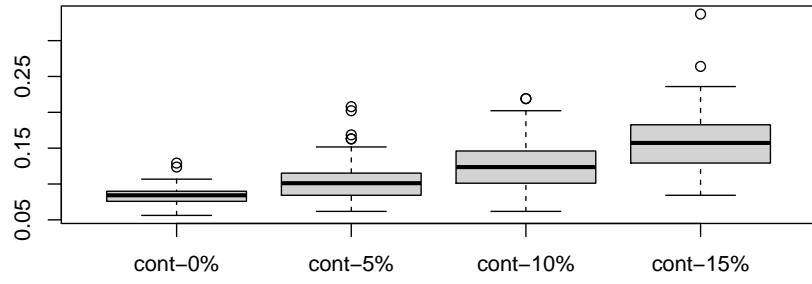
Logistic-LASSO

No. variables



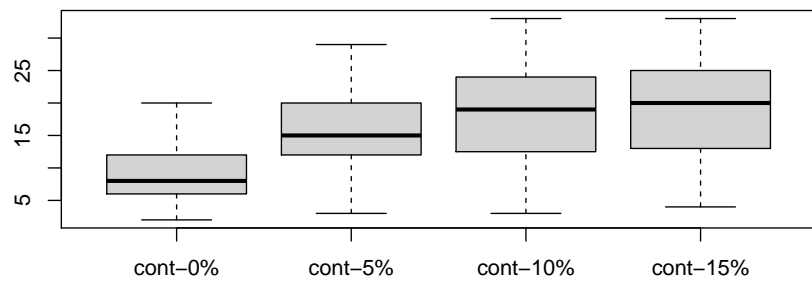
Logistic-LASSO

Misclassification error



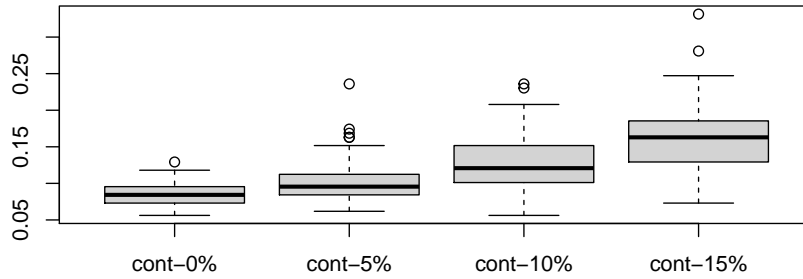
Logistic-SCAD

No. variables



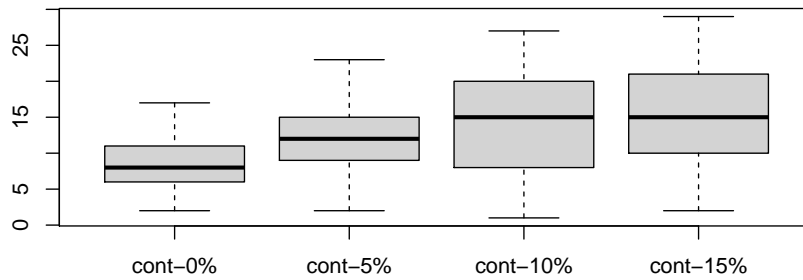
Logistic-SCAD

Misclassification error



Logistic-MCP

No. variables



Logistic-MCP

```
sessionInfo()
## R version 4.0.3 (2020-10-10)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Ubuntu 18.04.5 LTS
##
## Matrix products: default
## BLAS: /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.7.1
## LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.7.1
##
## locale:
## [1] LC_CTYPE=en_US.UTF-8 LC_NUMERIC=C
## [3] LC_TIME=en_US.UTF-8 LC_COLLATE=en_US.UTF-8
## [5] LC_MONETARY=en_US.UTF-8 LC_MESSAGES=en_US.UTF-8
## [7] LC_PAPER=en_US.UTF-8 LC_NAME=C
## [9] LC_ADDRESS=C LC_TELEPHONE=C
## [11] LC_MEASUREMENT=en_US.UTF-8 LC_IDENTIFICATION=C
##
## attached base packages:
## [1] stats graphics grDevices utils datasets
## [6] methods base
```

```
##
## other attached packages:
## [1] mpath_0.4-2.18 pamr_1.56.1 survival_3.2-7
## [4] cluster_2.1.0 glmnet_4.0 Matrix_1.2-18
## [7] gdata_2.18.0 knitr_1.30
##
## loaded via a namespace (and not attached):
## [1] magrittr_1.5 splines_4.0.3
## [3] MASS_7.3-51.6 doParallel_1.0.15
## [5] bst_0.3-20 pscl_1.5.5
## [7] gbm_2.1.5 lattice_0.20-41
## [9] foreach_1.5.0 stringr_1.4.0
## [11] tools_4.0.3 parallel_4.0.3
## [13] grid_4.0.3 gtable_0.3.0
## [15] xfun_0.19 iterators_1.0.12
## [17] gtools_3.8.2 numDeriv_2016.8-1.1
## [19] gridExtra_2.3 formatR_1.7
## [21] codetools_0.2-16 rpart_4.1-15
## [23] WeightSVM_1.7-5 shape_1.4.4
## [25] evaluate_0.14 stringi_1.4.6
## [27] compiler_4.0.3
```

References

Zhu Wang. MM for penalized estimation. 2019. URL <https://arxiv.org/abs/1912.11119>.